Math 0100

NAME: *Key*

You have ten minutes to answer the following question. You may not use calculators, notes, or other references. Please show your work and explain your answers; answers with no explanation will not receive full credit. *Good luck!*

Determine the interval of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{2^n}{9n}(7-x)^n$$

Ratio test: $\lim \left| \frac{2^{n+1}}{9(n+1)}(7-x)^{n+1} \cdot \frac{9n}{2^n (7-x)^n} \right| = \lim |2(7-x)| \left(\frac{n}{n+1}\right)^1$

$\Rightarrow \quad |2(7-x)| < 1$

$|7-x| < \frac{1}{2} \implies \quad 6.5 < x < 7.5$

@ endpoints?

@ 6.5 $\rightsquigarrow$ $\sum_{n=1}^{\infty} \frac{2^n}{9n}(\frac{1}{2})^n = \sum_{n=1}^{\infty} \frac{1}{9n}$ diverges p-series

@ 7.5 $\rightsquigarrow$ $\sum_{n=1}^{\infty} \frac{2^n (\frac{-1}{2})^n}{9n} = \sum \frac{(-1)^n}{9n}$ converges alternating series test.

$\Rightarrow$ interval is $\boxed{\begin{array}{c} 6.5 < x \le 7.5 \\[4pt] or \ x \in (6.5, 7.5] \end{array}}$